Noise induced oscillations in non-equilibrium steady state systems

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Abstract. We consider effect of stochastic sources upon self-organization process being initiated with creation of the limit cycle. General expressions obtained are applied to the stochastic Lorenz system to show that offset from equilibrium steady state can destroy the limit cycle at certain relation between characteristic scales of temporal variation of principle variables. Noise induced resonance related to the limit cycle is found analytically to appear in non-equilibrium steady state system if the fastest variations displays a principle variable, which is coupled with two different degrees of freedom or more.

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1. Introduction

Interplay between noise and non-linearity of dynamical systems [1] is known to arrive at crucial changing in behavior of systems displaying noise-induced [2, 3] and recurrence [4, 5] phase transitions, stochastic resonance [6, 7], noise induced pattern formation [8, 9], noise induced transport [10, 3] et cetera (see Ref. [11], for review). The constructive role of noise on dynamical systems includes hopping between multiple stable attractors [12, 13] and stabilization of the Lorenz attractor near the threshold of its formation [14, 15]. Such type behavior is inherent in finite systems which involve discrete entities (for instance, in ecological systems individuals form population stochastically in accordance with random births and deaths). Examples of substantial alteration of finite systems under effect of intrinsic noises give epidemics [16]–[18], predator-prey population dynamics [19, 20], opinion dynamics [21], biochemical clocks [22, 23], genetic networks [24], cyclic trapping reactions [25], etc.

Within the phase-plane language, phase transitions pointed out present the simplest case, where a fixed point appears only. We are interested in studying more complicated situation, when the system under consideration may display oscillatory behavior related to the limit cycle appearing as a result of the Hopf bifurcation [26, 27]. It has long been conjectured [28] that in some situations the influence of noise would be sufficient to produce cyclic behavior [29]. Recent consideration [30] allows the relation between the stochastic oscillations in the fixed point phase and the oscillations in the limit cycle phase to be elucidated. Moreover, excitable [31], bistable [32] and close to bifurcations [33] systems display oscillation behavior, whose adjacency to ideally periodic signal depends resonantly on the noise intensity [34], that was a reason to call these oscillations coherence resonance [31] or stochastic coherence [11]. Control of the coherence resonance regime was shown to be achieved with a time-delayed feedback, which enables to increase or decrease the regularity of motion [35]. Characteristically, a quasioscillatory behavior may be organized without any input signal, provided a stochastic nonlinear system has itself an intrinsic time scale. If this scale is driven by a multiplicative noise, which induce bistable behavior in a deterministically monostable medium, then a doubly stochastic resonance arises [36].

The simplest way to formulate the model related to systems with finite number $N < \infty$ of constituents is to consider the sum $\vec{S} = \sum_{i=1}^{N} \vec{\xi}_i$ of random state vectors $\vec{\xi}_i$ with components ξ_i^α, $\alpha = 1, \ldots, d$. Then, the state vector

$$\vec{S} = N\vec{X} + \sqrt{N}\vec{x} \tag{1}$$

is decomposed into a deterministic component being proportional to total system size N and a random one to be proportional to its square root [37]. In the limit of infinite particle numbers $N \to \infty$, such systems are faithfully described by deterministic equations to find time dependence $\vec{X}(t)$, which addresses the behavior of the system on a mean-field level. On the other hand, a systematic study of corrections due to finite system size can capture the behavior of fluctuations $\vec{x}(t)$ about the mean-field solution. These fluctuations are governed with the Langevin equations, however, in

difference of approach [30], we consider multiplicative noises instead of additive ones, on the one hand, and nonlinear forces instead of linear ones, on the other. Within such framework, the aim of the present paper is to extend analytical descriptions [30] of finite-size stochastic effects to non-equilibrium systems where noises play a crucial role with respect to periodic limit cycle solution creation or its supression. We will show that character of the stationary behavior of non-equilibrium system is determined by relation between scales of temporal variation of principle variables as well as their coupling. In contrast to the doubly stochastic resonance [36], we consider the case when multistable state is caused by both multiplicative noise and offset from equilibrium state.

The paper is organized along the following lines. In Section 2, we obtain conditions of the limit cycle creation using pair of stochastic equations with nonlinear forces and multiplicative noises. Sections 3, 4 are devoted to consideration of these conditions on the basis of stochastic Lorenz system with different regimes of principle variables slaving. According to Section 3 the limit cycle is created only in the case if the most fast variation displays a principle variable, which is coupled nonlinearly with two other degrees of freedom or more. Opposite case is studied in Section 4 to show that the limit cycle disappears in non-equilibrium steady state. Section 5 concludes our consideration.

2. Statistical picture of limit cycle

According to the theorem of central manifold [26], to achieve a closed description of a limit cycle it is enough to use only two variables x_α, $\alpha = 1, 2$. In such a case, stochastic evolution of the system under investigation is defined by the Langevin equations [39]

$$\dot{x}_\alpha = f^{(\alpha)} + \mathcal{G}_\alpha \zeta_\alpha(t), \quad \alpha = 1, 2 \tag{2}$$

with forces $f^{(\alpha)} = f^{(\alpha)}(x_1, x_2)$ and noise amplitudes $\mathcal{G}_\alpha = \mathcal{G}_\alpha(x_1, x_2)$, being functions of stochastic variables x_α, $\alpha = 1, 2$; white noises $\zeta_\alpha(t)$ are determined by usual conditions $\langle \zeta_\alpha(t) \rangle = 0$, $\langle \zeta_\alpha(t) \zeta_\beta(t') \rangle = \delta_{\alpha\beta} \delta(t - t')$. Within the assumption that microscopic transfer rates are non correlated for different variables x_α (see below), the probability distribution function $\mathcal{P} = \mathcal{P}(x_1, x_2; t)$ is determined by the Fokker-Planck equation

$$\frac{\partial \mathcal{P}}{\partial t} + \sum_{\alpha=1}^{2} \frac{\partial J^\alpha}{\partial x_\alpha} = 0, \tag{3}$$

where components of the probability current take the form

$$J^{(\alpha)} \equiv \mathcal{F}^{(\alpha)} \mathcal{P} - \frac{1}{2} \sum_{\beta=1}^{2} \frac{\partial}{\partial x_\beta} \left(\mathcal{G}_\alpha \mathcal{G}_\beta \mathcal{P} \right) \tag{4}$$

with the generalized forces

$$\mathcal{F}^{(\alpha)} = f^{(\alpha)} + \lambda \sum_{\beta=1}^{2} \frac{\partial \left(\mathcal{G}_\alpha \mathcal{G}_\beta \right)}{\partial x_\beta}, \tag{5}$$

being determined with choice of the calculus parameter $\lambda \in [0, 1]$ (for Ito and Stratonovich cases, one has $\lambda = 0$ and $\lambda = 1/2$, respectively). Within the steady state,

the components of the probability current take constant values $J_0^{(\alpha)}$ and the system behaviour is defined by the following equations:

$$\frac{\partial}{\partial x_1}\left(\mathcal{G}_1^2\mathcal{P}\right)+\frac{\partial}{\partial x_2}\left(\mathcal{G}_1\mathcal{G}_2\mathcal{P}\right)-2\mathcal{F}^{(1)}\mathcal{P}=-2J_0^{(1)}, \tag{6}$$

$$\frac{\partial}{\partial x_1}\left(\mathcal{G}_1\mathcal{G}_2\mathcal{P}\right)+\frac{\partial}{\partial x_2}\left(\mathcal{G}_2^2\mathcal{P}\right)-2\mathcal{F}^{(2)}\mathcal{P}=-2J_0^{(2)}. \tag{7}$$

Multiplying the first of these equations by factor $\mathcal{G}_2$ and the second one by $\mathcal{G}_1$ and then subtracting results, we arrive at the explicit form of the probability distribution function as follows:

$$\begin{aligned}&\mathcal{P}\left(x_1,x_2\right)=\frac{J_0^{(1)}\mathcal{G}_2\left(x_1,x_2\right)-J_0^{(2)}\mathcal{G}_1\left(x_1,x_2\right)}{\mathcal{D}\left(x_1,x_2\right)},\\&\mathcal{D}\left(x_1,x_2\right)\equiv\left(\mathcal{G}_2\mathcal{F}^{(1)}-\mathcal{G}_1\mathcal{F}^{(2)}\right)\\&+\frac{1}{2}\left[\left(\mathcal{G}_1^2\frac{\partial\mathcal{G}_2}{\partial x_1}-\mathcal{G}_2^2\frac{\partial\mathcal{G}_1}{\partial x_2}\right)-\mathcal{G}_1\mathcal{G}_2\left(\frac{\partial\mathcal{G}_1}{\partial x_1}-\frac{\partial\mathcal{G}_2}{\partial x_2}\right)\right].\end{aligned} \tag{8}$$

This function diverges at condition

$$2\left(\mathcal{G}_1\mathcal{F}^{(2)}-\mathcal{G}_2\mathcal{F}^{(1)}\right)=\left(\mathcal{G}_1^2\frac{\partial\mathcal{G}_2}{\partial x_1}-\mathcal{G}_2^2\frac{\partial\mathcal{G}_1}{\partial x_2}\right)-\mathcal{G}_1\mathcal{G}_2\left(\frac{\partial\mathcal{G}_1}{\partial x_1}-\frac{\partial\mathcal{G}_2}{\partial x_2}\right), \tag{9}$$

that physically means appearance of a domain of forbidden values of stochastic variables x_α, which is bonded with a closed line of the limit cycle. Characteristically, such a line appears only if the denominator $\mathcal{D}(x_1,x_2)$ of fraction (8) includes even powers of both variables x_1 and x_2.‡

It is worth to note that the analytical expression (8) of the probability distribution function becomes possible due to the special form of the probability current (4), where effective diffusion coefficient takes the multiplicative form $\mathcal{D}_{\alpha\beta}=\mathcal{G}_\alpha\mathcal{G}_\beta$. In general case, this coefficient is known to be defined with the expression [40]

$$\mathcal{D}_{\alpha\beta}=\sum_{ab}I_{ab}g_\alpha^a g_\beta^b, \tag{10}$$

where kernel I_{ab} determines transfer rate between microscopic states a and b, whereas factors g_α^a and g_β^b are specific noise amplitudes of values x_α related to these states. We have considered above the simplest case, when the transfer rate $I_{ab}=I$ is constant for all microscopic states. As a result, the diffusion coefficient (10) takes the needed form $\mathcal{D}_{\alpha\beta}=\mathcal{G}_\alpha\mathcal{G}_\beta$ with cumulative noise amplitudes $\mathcal{G}_\alpha\equiv\sqrt{I}\sum_a g_\alpha^a$ and $\mathcal{G}_\beta\equiv\sqrt{I}\sum_b g_\beta^b$.

3. Noise induced resonance within Lorenz system

As the simplest and most popular example of the self-organization induced by the Hopf bifurcation, we consider modulation regime of spontaneous laser radiation, whose behaviour is presented in terms of the radiation strength E, the matter polarization P

‡ Archetype of closed curves presents the circle $x_1^2+x_2^2=1$.

and the difference of level populations S [37]. With accounting for stochastic sources related, the self-organization process of this system is described by the Lorenz equations

$$\begin{aligned}\tau_E\dot{E} &= [-E + a_E P - \varphi(E)] + g_E\zeta(t),\\ \tau_P\dot{P} &= (-P + a_P ES) + g_P\zeta(t),\\ \tau_S\dot{S} &= [(S_e - S) - a_S EP] + g_S\zeta(t).\end{aligned} \tag{11}$$

Here, overdot denotes differentiation over time t; $\tau_{E,P,S}$ and $a_{E,P,S} > 0$ are time scales and feedback constants of related variables, respectively; $g_{E,P,S}$ are corresponding noise amplitudes, and S_e is driven force. In the absence of noises ($g_E = g_P = g_S = 0$) and at relations $\tau_P, \tau_S \ll \tau_E$ between time scales, the system (11) addresses to limit cycle only in the presence of the nonlinear force [41]

$$\varphi(E) = \frac{\kappa E}{1 + E^2/E_n^2} \tag{12}$$

characterized with parameters $\kappa > 0$ and E_n. In this Section, we consider noise effect in the case of opposite relations $\tau_E \ll \tau_P, \tau_S$ of time scales, when periodic variation of stochastic variables becomes possible even at suppression of the force (12).

It is convenient further to pass to dimensionless variables t, ζ, E, P, S, g_E, g_P, g_S with making use of the related scales:

$$\begin{aligned}&\tau_P;\ \zeta_s = \tau_P^{-1/2};\ E_s = (a_P a_S)^{-1/2},\ P_s = (a_E^2 a_P a_S)^{-1/2},\ S_s = (a_E a_P)^{-1};\\ &g_E^s = (\tau_P/a_P a_S)^{1/2},\ g_P^s = (\tau_P/a_E^2 a_P a_S)^{1/2},\ g_S^s = \tau_P^{1/2}/a_E a_P.\end{aligned} \tag{13}$$

Then, equations (11) take the simple form§

$$\begin{aligned}\sigma^{-1}\dot{E} &= -E + P - \varphi(E) + g_E\zeta(t),\\ \dot{P} &= -P + ES + g_P\zeta(t),\\ (\varepsilon/\sigma)\dot{S} &= (S_e - S) - EP + g_S\zeta(t),\end{aligned} \tag{14}$$

where the time scale ratios

$$\sigma = \tau_P/\tau_E, \quad \varepsilon = \tau_S/\tau_E \tag{15}$$

are introduced. In the absence of the noises, the Lorenz system (14) is known to show the usual bifurcation in the point $S_e = 1$ and the Hopf bifurcation at the driven force [38, 37]

$$S_e = \frac{\tau_P}{\tau_E}\,\frac{\tau_E^{-1} + \tau_S^{-1} + 3\tau_P^{-1}}{\tau_E^{-1} - \tau_S^{-1} - \tau_P^{-1}}. \tag{16}$$

However, the noiseless limit cycle ($g_E = g_P = g_S = 0$) is unstable and the Hopf bifurcation arrives at the strange attractor only.

With switching on the noises, the condition $\tau_E \ll \tau_P$ allows for to put l.h.s. of the first equation (14) to be equal zero. Then, the radiation strength is expressed with the equality

$$E = P + g_E\zeta(t), (17) \tag{17}$$

§ These equations are reduced to the initial Lorenz form [38] if we set $X \equiv \sqrt{\sigma/\varepsilon}E$, $Y \equiv \sqrt{\sigma/\varepsilon}P$, $Z \equiv S_e - S$, $r \equiv S_e$, $b \equiv \sigma/\varepsilon$ and $g_E = g_P = g_S = 0$.

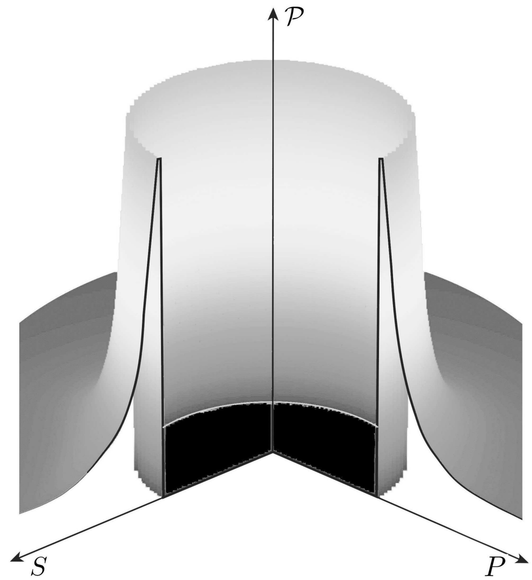

Figure 1. Steady state distribution function (8) at $J_0^{(P)} = 1$, $J_0^{(S)} = 10$, $\tau_P = \tau_S$, $S_e = 0.5$, $g_E = 0.5$, $g_P = 1.376$, $g_S = 2.5$

whose insertion into the system (14) reduces it into two-dimensional form

$$\dot{P} = -P(1-S) + \mathcal{G}_P\zeta(t),$$
$$\dot{S} = (\sigma/\varepsilon)\left[(S_e - S) - P^2\right] + \mathcal{G}_S\zeta(t) \tag{18}$$

with the effective amplitudes of multiplicative noises

$$\mathcal{G}_P = \sqrt{g_P^2 + g_E^2 S^2}, \quad \mathcal{G}_S = (\tau_P/\tau_S)\sqrt{g_S^2 + g_E^2 P^2} \tag{19}$$

and the generalized forces

$$\mathcal{F}^{(P)} = -P(1-S) + \lambda\frac{g_E^2}{\tau_S/\tau_P} S\sqrt{\frac{(g_S/g_E)^2 + P^2}{(g_P/g_E)^2 + S^2}},$$
$$\mathcal{F}^{(S)} = (\tau_P/\tau_S)\left[(S_e - S) - P^2\right] + \lambda\frac{g_E^2}{\tau_S/\tau_P} P\sqrt{\frac{(g_P/g_E)^2 + S^2}{(g_S/g_E)^2 + P^2}}. \tag{20}$$

In this way, the probability density (8) takes infinite values at condition

$$\left(\frac{g_S^2}{g_E^2} + P^2\right)\sqrt{\frac{g_P^2}{g_E^2} + S^2}P(1-S) + \left(\frac{g_P^2}{g_E^2} + S^2\right)\sqrt{\frac{g_S^2}{g_E^2} + P^2}\left[(S_e - S) - P^2\right]$$
$$+\frac{g_E^2}{2}\frac{\sigma}{\varepsilon}\left(\frac{g_S^2}{g_E^2} + P^2\right)^{\frac{3}{2}} S - \frac{g_E^2}{2}\left(\frac{g_P^2}{g_E^2} + S^2\right)^{\frac{3}{2}} P = 0, \tag{21}$$

where we choose the simplest case of the Ito calculus ($\lambda = 0$).

Reduced Lorenz system (18) has two-dimensional form (2), where the role of variables x_1 and x_2 play the matter polarization P and the difference of level populations S. According to the distribution function (8) shown in Fig.1, the stochastic variables P and S are realized with non-zero probabilities out off the limit cycle only, whereas in

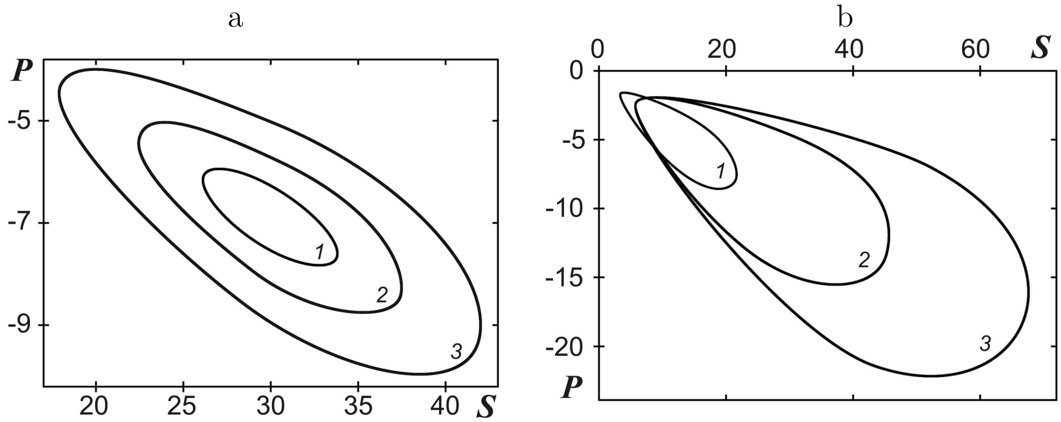

Figure 2. Form of limit cycle determined with Eq.(21) at $\varepsilon = 1$, $\sigma = 1$ and: a) $g_E = 0.5$, $g_P = 11$, $g_S = 6$ (curves 1-3 relate to $S_e = 0.5, 1.0, 2.0$, respectively); b) $S_e = 0.5$, $g_P = 7.5$, $g_S = 6.5$ (curves 1-3 relate to $g_E = 1.0, 0.6, 0.5$, respectively)

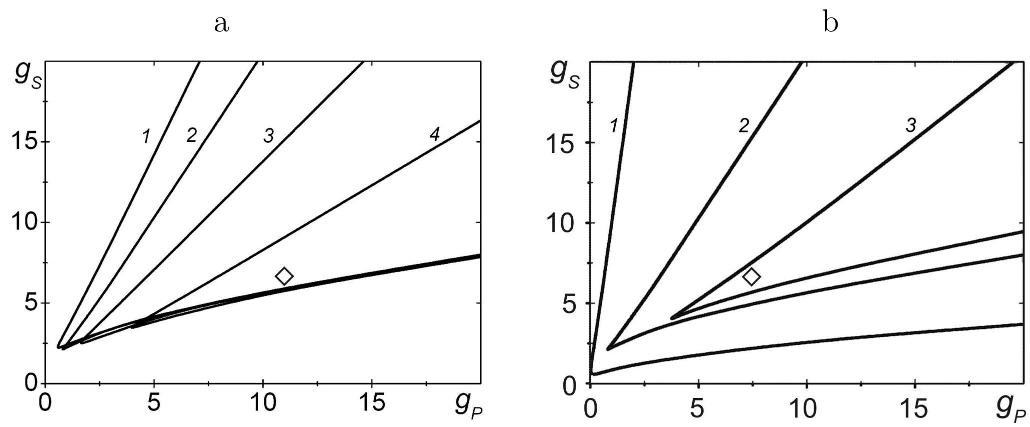

Figure 3. Phase diagrams of the limit cycle creation at $\varepsilon = 1$, $\sigma = 1$ and: a) $g_E = 0.5$, curves 1-4 correspond to $S_e = 0.0, 0.5, 1.0, 2.0$, respectively; b) $S_e = 0.5$, curves 1-3 correspond to $g_E = 0.1, 0.5, 1.0$, respectively (diamonds relate to the values g_P and g_S, for which limit cycles in Fig.2 are depicted)

its interior the domain of forbidden values P, S appears. That is principle difference from the deterministic limit cycle, which bounds a domain of unstable values of related variables. The form of this domain is shown in Fig.2 at different values of the noise amplitudes g_E, g_P, g_S and driven force S_e. It is seen, this domain grows with increase of the driven force S_e, whereas increase of the force fluctuations g_E shrinks it. On the other hand, phase diagrams depicted in Fig.3 show that increasing the noise amplitudes of both polarization and difference of level populations enlarges domain of the limit cycle creation (more exactly, the noise amplitude g_E shrinks this domain from both above and below, whereas increase of the driven force S_e makes the same from above only).

The principle peculiarity of the limit cycles obtained is that their form, determined with Eq.(21), does not depend on a non-equilibrium degree fixed by the stationary probability currents $J_0^{(1,2)}$, whereas the probability (8) itself does not equal zero at

conditions $J_0^{(1,2)} \neq 0$ only. In this connection, one should be pointed out a non-triviality of the problem of numerical solution of the reduced Lorenz system (18), which determines these limit cycles initially. Indeed, resolving this problem proposes the following steps: i) direct solution of the stochastic equations (18) to find a set of the time dependencies $P(t)$ and $S(t)$; ii) numerical determination of the time-dependent probability $\mathcal{P}(P, S; t)$ to realize entire set of possible solutions of the equations (18); iii) selection of non-equilibrium solutions, which obey to the steady state condition $J^{(\alpha)} = J_0^{(\alpha)}$, $\alpha = 1, 2$, determined with the probabillity current (4); iv) calculation of the probability distribution $\mathcal{P}_s(P, S)$ of the steady state solutions; v) determination of the stochastic limit cycle according to the condition $\mathcal{P}_s(P, S) = \infty$. Realization of this program is in progress.

4. Lorenz system without limit cycle

According to Ref. [41], at conditions $\tau_P \ll \tau_E, \tau_S$, the deterministic system ($g_{E,P,S} = 0$) has a limit cycle only at large intensity κ of non linear force (12). In this case, it is convenient to measure the time t in the scale τ_E and replace τ_P by τ_E in set of scales (13). Then, one obtains instead of Eq.(17) the relation

$$P = ES + g_P \zeta(t), \tag{22}$$

due to which the Lorenz system (14) is reduced to two-dimensional form

$$\begin{aligned} \dot{E} &= -\left[E(1 - S) + \varphi(E)\right] + \mathcal{G}_E \zeta(t), \\ \dot{S} &= \varepsilon^{-1}\left[S_e - S(1 + E^2)\right] + \mathcal{G}_S \zeta(t) \end{aligned} \tag{23}$$

with the effective noise amplitudes

$$\mathcal{G}_E = \sqrt{g_P^2 + g_E^2}, \quad \mathcal{G}_S = \varepsilon^{-1}\sqrt{g_S^2 + g_P^2 E^2}. \tag{24}$$

The generalized forces are as follows:

$$\begin{aligned} \mathcal{F}^{(E)} &= -\left[E(1 - S) + \varphi(E)\right], \\ \mathcal{F}^{(S)} &= \varepsilon^{-1}\left[(S_e - S) - SE^2\right] + \lambda \frac{g_P^2}{\varepsilon} E \sqrt{\frac{1 + (g_E/g_P)^2}{(g_S/g_P)^2 + E^2}}. \end{aligned} \tag{25}$$

The probability distribution function (8) diverges at condition

$$\begin{aligned} &\frac{(g_S/g_P)^2 + E^2}{1 + (g_E/g_P)^2}\left[\varphi(E) + E(1 - S)\right] \\ &+ \sqrt{\frac{(g_S/g_P)^2 + E^2}{1 + (g_E/g_P)^2}}\left[S_e - S(1 + E^2)\right]\left(\lambda - \frac{1}{2}\right) g_P^2 E = 0, \end{aligned} \tag{26}$$

being the equation, which does not include even powers of the variable S.

As a result, one can conclude that offset from equilibrium steady state destroys a deterministic limit cycle at the relations $\tau_P \ll \tau_E, \tau_S$ between characteristic scales. This conclusion is confirmed with Fig.4, which shows divergence of the probability distribution function on the limit cycle of variation of the radiation strength E and

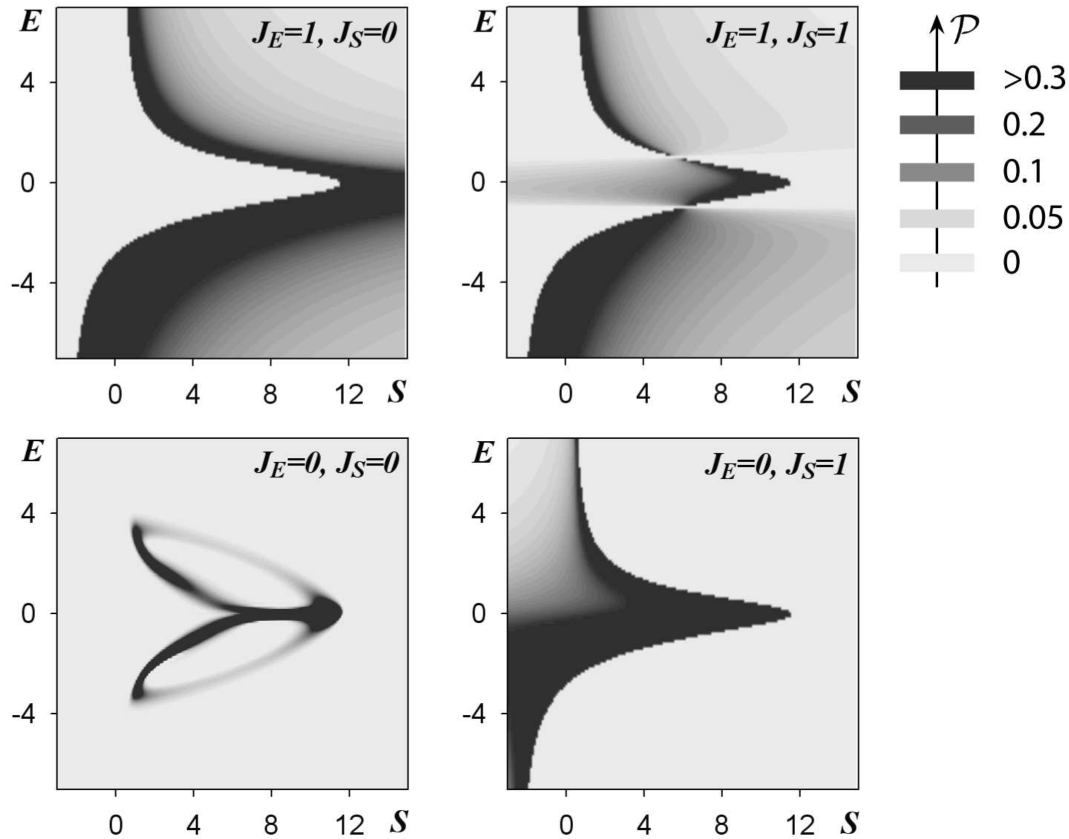

Figure 4. Steady state probability distribution function in dependence of the radiation strength E and the difference of level populations S at conditions $\tau_P \ll \tau_E = \tau_S$, $\kappa = 10$, $S_e = 11.6$, $g_E = 0.2$, $g_P = 0.2$, $g_S = 0.2$ and different probability currents $J_0^{(E)}$, $J_0^{(S)}$ (shown on the panels related)

the difference of level populations S at zeros probability currents $J_0^{(E)}$ and $J_0^{(S)}$ only. With increase of these currents the system escapes from equilibrium steady state and maximum of the distribution function shifts to non-closed curves to be determined with equation (26).

5. Conclusion

We have considered effect of stochastic sources upon self-organization process being initiated with creation of the limit cycle. In Sections 3, 4, we have applied general relations obtained in Section 2 to the stochastic Lorenz system. We have shown that offset from equilibrium steady state can destroy or create the limit cycle in dependence of relation between characteristic scales of temporal variation of principle variables.

Investigation of the Lorenz system with different regimes of principle variables slaving shows that additive noises can take multiplicative character if one of these noises has many fewer time scale than others. In such a case, the limit cycle may be created if the most fast variable is coupled with more than two slow ones. However,

the case considered in Section 4 shows that such dependence is not necessary to arrive at limit cycle. The formal reason is that within adiabatic condition $\tau_P \ll \tau_E$ both noise amplitude $\mathcal{G}_S(E)$ and generalized force $\mathcal{F}^{(S)}(E)$, determined with Eqs. (24) and (25), enclose the squared strength E^2, but do not include the square S^2.

The limit cycle is created if the fastest variations displays a principle variable, which is coupled with two different degrees of freedom or more. Indeed, at the relations $\tau_E \ll \tau_P, \tau_S$ of relaxation times considered in Section 3, the strength E evolves according to the stochastic law of motion (17). Accounting this relation in the nonlinear terms of two last equations (14) arrives at dependencies of the noise amplitudes of the polarization P and the difference of level populations S on both variables S and P themselves. Due to gaussian nature of the noises their variances are additive values [39, 40], so that effective noise amplitudes $\mathcal{G}_P$ and $\mathcal{G}_S$ of the principle variables are defined by Eqs. (19), which include both squares S^2 and P^2. As a result, solutions of Eq.(21) become double-valued to be related to the limit cycle.

This cycle appears physically as stochastic coherence, that has been observed both numerically [14, 31] and analytically [42]. Analogously to the doubly stochastic resonance [36], such a resonance may be organized if stochastic nonlinear system has two noises, but both of them must be multiplicative in nature. Moreover, the system under study acquires an intrinsic time scale related to multistable state only far off equilibrium statistical state. In opposite to the deterministic limit cycle which bounds a domain of unstable values of related variables, in our case stochastic variables evolve out off the limit cycle only, whereas in its interior the domain of forbidden values appears.

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